[2] Townsend, A. A., "On the Fine-Scale Structure of Turbulence," *Proceedings of the Royal Society*, Vol. A208, 1951, pp. 534–542.

[3] Corrsin, S., "Further Generalization of Onsager's Cascade Model for Turbulent Spectra," *The Physics of Fluids*, Vol. 7, No. 8, Aug. 1964, pp. 1156–1159.

[4] Hinze, J. O., *Turbulence*, McGraw-Hill, New York, 1959, pp. 171–173, 200.

[5] Grant, H. L., Hughes, B. A., Vogel, W. M., and Moilliet, A., "The Spectrum of Temperature Fluctuations in Turbulent Flow," *Journal of Fluid Mechanics*, Vol. 34, Pt. 3, 1968, pp. 423–442.

[6] Taylor, G. I., "Statistical Theory of Turbulence," *Proceedings of the Royal Society of London*, Vol. 151A, 1935, pp. 421–478.

[7] Grant, H. L., Stewart, R. W., and Moilliet, A., "Turbulence Spectra from a Tidal Channel," *Journal of Fluid Mechanics*, Vol. 12, 1962, pp. 241–268.

[8] Gibson, C. H., Chen, C. C., and Lin, S. C., "Measurements of Turbulent Velocity and Temperature Fluctuations in the Wake of a Sphere," *AIAA Journal*, Vol. 6, No. 4, April 1968, pp. 642–649.

[9] Gibson, C. H. and Schwarz, W. H., "The Universal Equilibrium Spectra of Turbulent Velocity and Scalar Fields," *Journal of Fluid Mechanics*, Vol. 16, 1963, pp. 365–384.

[10] Uberoi, M. S. and Freymuth, P., "Spectra of Turbulence in Wakes behind Circular Cylinders," *The Physics of Fluids*, Vol. 12, No. 7, July 1969, pp. 1359–1363.

[11] Batchelor, G. K., *The Theory of Homogeneous Turbulence*, Cambridge University Press, London, 1960, Chap. 6.

[12] Kistler, A. L. and Vrebralovich, T., "Grid Turbulence at Large Reynolds Numbers," *Journal of Fluid Mechanics*, Vol. 26, Pt. 1, 1966, pp. 37–47.

[13] Shkarofsky, I. P., "Time Variations of Generalized Spectral Functions Turbulence," Research Rept. 7-801-71, Sept. 1968, RCA Limited Research Labs., Montreal, Canada.

Three-Dimensional Stagnation-Point Heat Transfer in Equilibrium Air Flows

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Nomenclature

C = $\rho\mu/(\rho\mu)_\infty$
f = stream function such that $\partial f/\partial\eta = (u_i/u_{i\infty})$; $i = 1,2$
g = enthalpy function = h/h_∞
h = enthalpy
K = transverse to principal inviscid velocity ratio = α_2/α_1
Nu = Nusselt number = $q_s Pr_s x_1/(h_\infty - h_s)\mu_s$
Pr = Prandtl number, frozen
q = heat flux normal to the surface
Re = Reynolds number = $\rho_s\alpha_1 x_1^2/\mu_s$
u = component of velocity
U = airspeed
x = coordinate
α = inviscid velocity gradient, $u_i = \alpha_i x_i f_i$
λ = heat transfer parameter = $C_s g_s'/(1 - g_s)Pr_s$
μ = viscosity
η = transformed coordinate, $\eta = \left(\dfrac{\rho_\infty\alpha_1}{\mu_\infty}\right)^{1/2}\displaystyle\int_0^{x_3}(\rho/\rho_\infty)d\tilde{x}_3$
ρ = density

Subscripts

i = orthogonal coordinate system directions with 1 and 2 being along the surface and 3 normal to it
s = surface conditions
∞ = outer limit of the boundary layer

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THIS Note summarizes the results of a parametric study of heat transfer to general, regular three-dimensional stagnation points in equilibrium air flows at speeds up to 29,000 fps. The boundary-layer flows considered here are found at stagnation points of a class of three-dimensional bodies ranging from spheres, through cylinders, to saddle shapes with equal magnitudes of inviscid velocity gradients in the two principal planes. The main objective here is to present engineering data that the previous studies of this class of problems failed to produce. The most extensive published data are those of: 1) Poots[1] who considered gases with $\rho\mu$ = constant, $\rho \propto h^{-1}$, $Pr = 1.0$; and 2) Libby[2] who extended the work of Poots to $Pr = 0.7$ and surface mass transfer.

In both these studies it seems that the methods of solution influenced the decision to employ the simplifying assumption of $\rho\mu$ = const, thus, in effect, eliminating a critical heat-transfer variable and therefore producing data of limited engineering value. On the other hand Reshotko[3] obtained useful engineering estimates by employing a transformation that enabled him to use existing two-dimensional results. By developing simple, accurate engineering relations the present effort follows the pioneering work of Fay and Riddell,[4] who presented an expression for real-gas heat transfer to an axisymmetric stagnation point, and the later work of Cohen,[5] who developed correlation functions for a variety of laminar boundary-layer flows.

Governing Equations and Method of Solution

The basic differential equations describing compressible variable-properties three-dimensional boundary-layer flows have been discussed thoroughly elsewhere (e.g., Chan[6]) so that only their final form will be shown here:

$$(Cf_1'')' + (f_1 + Kf_2)f_1'' + (\rho_\infty/\rho - f_1'^2) = 0 \quad (1a)$$

$$(Cf_2'')' + (f_1 + Kf_2)f_2'' + K(\rho_\infty/\rho - f_2'^2) = 0 \quad (1b)$$

$$(Cg'/Pr)' + (f_1 + Kf_2)g' = 0 \quad (1c)$$

Table 1 Heat-transfer parameter λ

Airspeed, fps		K					
	g_s	1.0	0.6	0.2	0.0	−0.3	−0.5
5 × 10³	0.30	0.996	0.892	0.784	0.730	0.666	0.663
5	0.50	0.981	0.880	0.775	0.724	0.668	0.669
5	0.70	0.965	0.865	0.763	0.715	0.668	0.670
5	0.90	0.950	0.852	0.753	0.707	0.665	0.668
10 × 10³	0.10	0.986	0.884	0.776	0.723	0.658	0.655
10	0.20	0.960	0.860	0.756	0.706	0.648	0.647
10	0.30	0.939	0.842	0.741	0.693	0.639	0.640
10	0.50	0.919	0.824	0.727	0.681	0.632	0.636
15 × 10³	0.05	0.990	0.887	0.779	0.725	0.661	0.657
15	0.10	0.973	0.873	0.767	0.715	0.654	0.651
15	0.20	0.956	0.857	0.754	0.704	0.647	0.647
15	0.30	0.947	0.850	0.748	0.600	0.645	0.647
20 × 10³	0.05	0.995	0.892	0.783	0.730	0.666	0.661
20	0.10	0.984	0.882	0.775	0.722	0.661	0.658
20	0.20	0.974	0.874	0.768	0.717	0.659	0.658
20	0.30	0.969	0.869	0.765	0.715	0.660	0.660
25 × 10³	0.01	1.020	0.914	0.802	0.747	0.679	0.673
25	0.05	1.006	0.902	0.764	0.738	0.673	0.668
25	0.10	1.000	0.896	0.788	0.734	0.671	0.668
25	0.20	0.993	0.891	0.783	0.731	0.672	0.669
25	0.30	0.989	0.887	0.781	0.729	0.672	0.672
29 × 10³	0.01	1.026	0.920	0.807	0.752	0.684	0.677
29	0.05	1.015	0.910	0.799	0.744	0.674	0.672
29	0.10	1.010	0.905	0.795	0.741	0.678	0.674
29	0.20	1.004	0.900	0.791	0.739	0.678	0.676
29	0.30	0.999	0.896	0.788	0.737	0.679	0.679

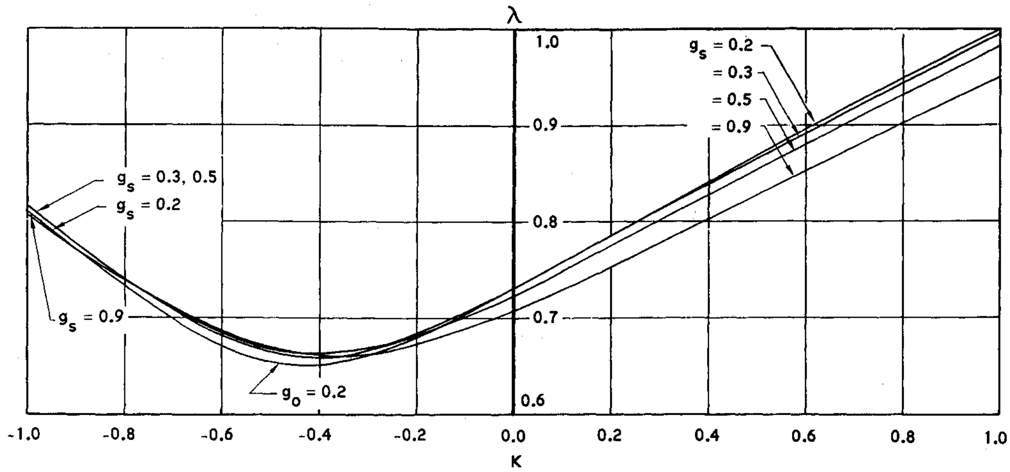

Fig. 1 Heat-transfer parameter λ, U = 5000 fps.

The boundary conditions are

$$\eta = 0, f_i' = f_i = 0, g = g_s, i = 1,2 \quad (2a)$$

$$\eta \to \infty, f_i' \to 1, g \to 1.0, i = 1,2 \quad (2b)$$

The prime denotes differentiation with respect to η.

The relations for density and C were taken from Cohen[5] and the variation of the frozen Prandtl number with enthalpy was calculated using the three polynomial curve fits of Clutter and Smith.[7] Lewis number was taken to be unity since Cohen showed that the effects of nonunity Lewis number on heat transfer were minor and could be estimated quite accurately. The range of air properties correlations are stated by Cohen to be between $300K$ and flight velocities corresponding to 29,000 fps. The method of solution employed here was developed in Ref. 8 where it was applied to a wide range of boundary-layer problems. A typical example with 151 steps across the boundary layer converged to 4 decimal places in under 4 sec of IBM 360/65 time.

Results

It is convenient to define a heat-transfer parameter that contains the bulk of the effect of the variation of gas properties across the boundary layer. With the definition

$$\lambda = C_s g_s'/(1 - g_s)Pr_s = (Nu/Re^{1/2})(C_s^{1/2}/Pr_s) \quad (3)$$

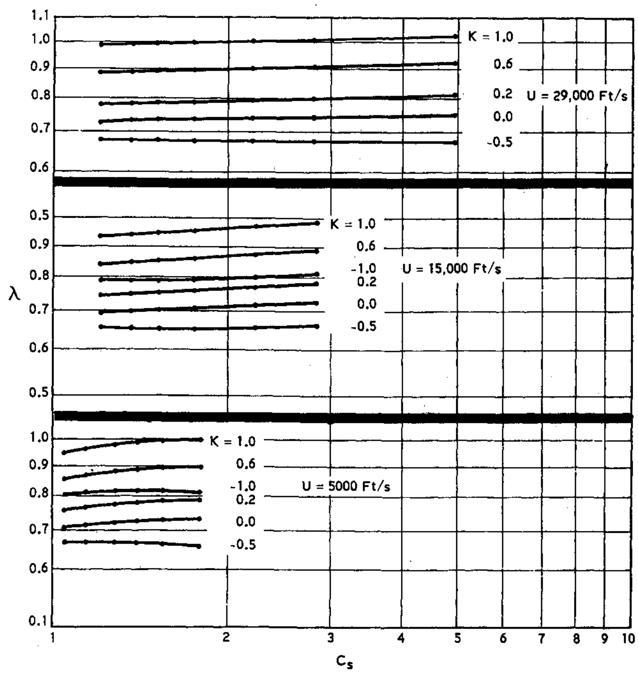

Fig. 2 Heat-transfer parameter λ.

we have the usual grouping of the Nusselt number divided by the square root of Reynolds number, together with some further accounting for real-gas effects. The heat-transfer results are summarized in Table 1.

The variation of λ with K and g_s is quite similar for all energy levels and the curves shown in Fig. 1 are typical of those found in the whole speed range. The weak influence of g_s on λ should be noted.

It is quite significant that the influence of shape, or cross-flow, increases with decreasing g_s, and the greatest variation in λ is observed at the lowest values of g_s. This result does not support the frequently employed small-cross-flow assumption which uncouples the transverse and principal plane momentum equations for flows with "cold walls."

It is customary in heat-transfer work to account for the variation of the physical properties of the gas by correlating the heat transfer coefficients with some values of the density-viscosity product C in the boundary layer. At axisymmetric and cylindrical stagnation points, the surface value C_s was used successfully in Refs. 4 and 5 to correlate data for a wide range of airspeeds and g_s. The weak influence of U and g_s on λ is exhibited in Fig. 2. Quite obvious from this figure is the strong influence of shape, characterized by K, which enters the problem through the acceleration terms and thus involves the density of the boundary-layer gas. In order to simplify the correlation functions it is convenient to account separately for the effects of the two main parameters of the problem. One of the more obvious approaches is to normalize the real gas data by the results for gases in which the variations of C and Pr are suppressed.

The results of Ref. 8 for a model gas with $\rho \propto h^{-1}$, $C = 1.0$, and $Pr = 0.7$ are shown in Table 2 to illustrate the effects of g_s without any variation of C_s or Pr. When the real-gas data are normalized by the model gas results, then

$$\lambda^* = \lambda/\lambda_{(C=1,Pr=0.7)} \quad \text{(same } g_s\text{)} \quad (4)$$

is virtually independent of K.

All the data for $1 \leq C \leq 5$ are correlated to within 4% with the relation

$$\lambda^* - 1 = 0.12(C_s - 1)^{0.5} \quad (5)$$

This relation is easier to use than may appear at first glance

Table 2 Heat-transfer parameter $\lambda_{(C=1,Pr=0.7)}$

	$g_s = h/h_\infty$					
K	0.01	0.1	0.2	0.4	0.6	0.8
1.0	0.8667	0.8751	0.8854	0.9035	0.9203	0.9360
0.6	0.7760	0.7843	0.7933	0.8100	0.8255	0.8485
0.2	0.6768	0.6853	0.6944	0.7111	0.7265	0.7405
0.0	0.6243	0.6339	0.6440	0.6623	0.6790	0.6945
−0.5	0.5362	0.5547	0.5728	0.6035	0.6295	0.6525
−0.75	0.6058	0.6203	0.6353	0.6623	0.6890	0.7110
−1.0	0.6987	0.7130	0.6601	0.7540	0.7865	0.8075

because the data of Table 2 may be approximated quite accurately by straight-line relations.

With such straight-line fits of data for a given value of K, it seems possible to correlate the data to an accuracy that is well within the accuracy of the physical properties equations and the assumption of Lewis number of unity. Corrections for Lewis number $\neq 1$ which are discussed in Ref. 5 will improve the accuracy of the desired relation to the point where further refinements may not be worthwhile.

References

[1] Poots, G., "Compressible Laminar Boundary-Layer Flow at a Point of Attachment," *Journal of Fluid Mechanics*, Vol. 22, Pt. 1, 1965, pp. 197–208.

[2] Libby, P. A., "Heat and Mass Transfer at a General Three-Dimensional Stagnation Point," *AIAA Journal*, Vol. 7, No. 3, March 1967, pp. 507–517.

[3] Reshotko, E., "Heat Transfer to a General Three-Dimensional Stagnation Point," *Jet Propulsion*, Vol. 28, No. 1, 1958, pp. 58–60.

[4] Fay, J. A. and Riddell, F. R., "Theory of Stagnation Point Heat Transfer in Dissociated Air," *Journal of the Aeronautical Sciences*, Vol. 25, No. 2, Feb. 1958, pp. 73–88.

[5] Cohen, N. B., "Boundary-Layer Similar Solutions and Correlation Equations for Laminar Heat Transfer Distribution in Equilibrium Air at Velocities up to 41,100 Feet per Second," TR-118, 1961, NASA.

[6] Chan, Y. Y., "A Note in a Similarity Transformation for Three-Dimensional Compressible Laminar Boundary-Layer Equations," Aeronautics Rept. LR-469, Nov. 1967, National Aeronautical Establishment, Ottawa, Canada.

[7] Clutter, D. W. and Smith, A. M. O., "Solution of the General Boundary-Layer Equations for Compressible Laminar Flow Including Transverse Curvature," Rept. LB-31088, 1963, Douglas Corp., Long Beach, Calif.

[8] Wortman, A., "Mass Transfer in Self-Similar Boundary-Layer Flows," Rept., Aug. 1969, Northrop Corp., Hawthorne, Calif.; also, Ph.D. dissertation, Aug. 1969, Dept. of Engineering and Applied Science, Univ. of California, Los Angeles, Calif.

Viscous Blunt-Body Flow with Radiation

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Problem and Results

THE following analysis is intended to clarify the controversial question[1,2] how radiative emission effects the flow in the shock structure of a low-density hypersonic flow around a blunt body. Bush's treatment[3] of the viscous blunt-body problem by the method of matched asymptotic expansions is extended to the flow of a radiating gas. The result is that the radiation term of the energy equation is a small, higher-order term in the entire shock structure (consisting of three different regions) if the radiation-convection parameter Γ † is of order one; in the shock layer, however, the radiation term is then of the same order of magnitude as the largest convection term. It is, therefore, correct to omit the radiation term in the shock structure provided that the limiting case of very strong radiation is excluded. Consequently Cheng's modified shock relations[4] may be used as outer boundary conditions for the viscous and radiating shock layer, and the shock layer equations can be solved independently of the solution of the shock structure.[5,6] This does not mean, however, that there is no first-order effect of radiation on the shock structure; for, an indirect influence of radiation on the shock structure arises from matching the shock structure solution with the shock layer solution.

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† Γ is defined subsequent to Eq. (1).

Analysis

Nondimensional variables are used, the notation being the same as in Bush's paper[3]; x and y are boundary-layer coordinates, u and v are the velocity components in x and y direction, r is the distance from the axis of symmetry, $\varkappa$ the longitudinal curvature of the axisymmetric body, Φ the inclination angle of the body surface, and $h = 1 + \varkappa y$. All lengths are referred to the body nose radius a, all flow quantities to their freestream values. A perfect gas is assumed, having constant specific heats, constant Prandtl number Pr (of order 1) and viscosity μ varying as T^{ω}. The equations of continuity and momentum are not changed by radiative energy transfer, but the equation of energy, as given by Bush,[3] has to be completed by adding a radiation term. Because of the low density, absorption within the shock layer and shock structure is negligible.[6] The energy emitted per unit time and unit mass is denoted by Q^*, and a nondimensional emission rate $Q = Q^*/Q_{\max}^*$ is introduced, with $Q_{\max}^*$ being the maximum value of Q^* in the flowfield. Now the energy equation can be written as

$$\rho\left(\frac{u}{h}\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right) - \frac{2\epsilon}{1+\epsilon}\left(\frac{u}{h}\frac{\partial p}{\partial x} + v\frac{\partial p}{\partial y}\right) = \frac{1}{PrRe}\left[\frac{\partial}{\partial y}\left(\mu\frac{\partial T}{\partial y}\right) + \left(\frac{\varkappa}{h} + \frac{\cos\Phi}{r}\right)\mu\frac{\partial T}{\partial y} + \frac{1}{h}\frac{\partial}{\partial x}\left(\frac{\mu}{h}\frac{\partial T}{\partial x}\right) + \frac{\sin\Phi}{r}\frac{\mu}{h}\frac{\partial T}{\partial x}\right] + \frac{2\epsilon M^2\mu}{(1-\epsilon)Re}\times \left[2\left(\frac{\partial v}{\partial y}\right)^2 + 2\left(\frac{1}{h}\frac{\partial u}{\partial x} + \frac{\varkappa v}{h}\right)^2 + 2\left(\frac{u\sin\Phi + v\cos\Phi}{r}\right)^2 + \left(\frac{\partial u}{\partial y} - \frac{\varkappa u}{h} + \frac{1}{h}\frac{\partial v}{\partial x}\right)^2 - \frac{2}{3}\left(\frac{1}{h}\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\varkappa v}{h} + \frac{u\sin\Phi + v\cos\Phi}{r}\right)^2\right] - \frac{2\epsilon M^2}{1-\epsilon}\Gamma\rho Q \quad (1)$$

where M is the freestream Mach number, Re the freestream Reynolds number, $Re = \rho_\infty U_\infty a/\mu_\infty$, Γ a radiation-convection parameter, $\Gamma = aQ_{\max}^*/U_\infty^3$,‡ and $\epsilon = (\gamma - 1)/(\gamma + 1)$ with γ as ratio of the specific heats.

For the case without radiation, $\Gamma = 0$, asymptotic expansions were constructed by Bush[3] for $M \to \infty$, $Re \to \infty$, and $\epsilon \to 0$ in such a way that $\delta = (1 - \epsilon)/2\epsilon M^2 \to 0$. In order to extend the expansions to the flow of a radiating gas, we notice that the orders of magnitude of the flow variables will not be changed by the effects of radiation, provided that the case of very strong radiation ($\Gamma \to \infty$) is excluded from the considerations. Hence we can take over Bush's matched asymptotic expansions into the analysis of the radiating field, and the extension is made by simply expanding the emission term of the energy equation (1) in the same manner as the flow variables. In the course of these developments, it will be assumed that Q^*, i.e., the energy emission per unit mass, decreases with decreasing temperature at least as $T^{1-\omega}$, and that Q^* also decreases with decreasing density or that is almost independent of the density. These assumptions are usually valid for nonequilibrium radiation[6] as well as in the case of local thermodynamic equilibrium.[8] The results of the expansions for the various flow regions are given below, and the magnitude of the radiation term, denoted by q_{rad}, is compared with the magnitude of a typical leading term, which may be either a convection term q_{conv}, or a conduction term q_{cond}.

‡ If the gas were in local thermodynamic equilibrium, Γ would be equivalent to the parameter $\Gamma_s\alpha_s\epsilon a$ used by Liu and Sogame.[7]